Behaviour of a Reduced Basis method in a bifurcation thermoconvective problem

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The reduced basis approximation is a discretization method that can be implemented for solving of parameter-dependent problems $\mathcal{P}(\phi(\mu),\, \mu) = 0$ with parameter μ in cases of many queries. This method consists in approximating the solution $\phi(\mu)$ of $\mathcal{P}(\phi(\mu),\, \mu) = 0$ by a linear combination of *appropriate* preliminary computed solutions $\phi(\mu_i)$ with $i = 1, 2, ...N$ such that μ_i are parameters chosen by an iterative procedure using the *kolmogorov n-width* measures [2, 4].

In this work [1] it is applied to a two dimensional Rayleigh-Bénard problem with constant viscosity that depends on the Rayleigh number, $\mathcal{P}\,(\phi(R),\, R) = \vec{0}$.

Multiple steady solutions can be found for different Rayleigh numbers and stable solutions coexist at the same values of external physical parameters [3]. The reduced basis method permits to speed up the computations of these solutions at any value of the Rayleigh number chosen in a fixed interval associated with a single bifurcation branch while maintaining accuracy.

The problem is numerically solved by the Galerkin variational formulation using the Legendre Gauss-Lobatto quadrature formulas together with the reduced basis $\{\phi(R_i), i = 1, 2, ...N\}$ such that $\phi(R) \sim \sum_{i=1}^{N} \lambda_i \phi(R_i)$.

References

[1] H. Herrero, Y. Maday and F. Pla. *RB (Reduced basis) for RB (Rayleigh-Bénard)*. Computer Methods in Applied Mechanics and Engineering (to appear) (2013).

[2] Y. Maday, A.T. Patera and G. Turinici. *Convergence theory for reduced-basis approximations of single-parameter elliptic partial differential equations*, J. Sci. Comput. **17**, no. 1-4, 437-446 (2002).

[3] F. Pla, A.M. Mancho and H.Herrero. *Bifurcation phenomena in a convection problem with temperature dependent viscosity at low aspect ratio*. Physica D **238** , 572-580 (2009).

[4] C. Prud'homme, D.V. Rovas, K. Veroy, L. Machiels, Y. Maday, A.T. Patera and G. Turinici. *Reliable real-time solution of parametrized partial differential equations: Reduced-basis output bound methods*. Journal of Fluids Engineering, **124** (1), 70-80 (2002).

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