

# Examples of emergence of a periodic complexity in biological evolution

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It is known that population dynamics enables us to understand certain features of biological evolution, in particular the preservation and even the emergence of biological diversity. Indeed, the stability of the attractors of the corresponding dynamical systems constitute a more sure criterion of evolution than the classical “survival of more adapted” (which appears as ambiguous and not fitted to the very structure of the involved phenomena). Starting from the classical Lotka-Volterra’s predator/prey model, and more precisely from its Kolmogorov’s non-linear version (incorporating terms ensuring the uniqueness and stability of a periodic cycle), we address analogous situations with two predators and one prey. When each predator enjoys an advantage with respect to the other one and these advantages are of different nature (i. e. having an incidence on the differential equations described by different nonlinear terms), a periodic cycle involving the three species emerges (= preservation of the diversity) for certain values of the parameters.

This talk is concerned with examples of the previous situation when one of the predators has a higher natality than the other, which, oppositely, enjoys a better capability to take benefit of scarce preys. There is a stable periodic cycle, but the most interesting feature is the very long and entangled transient. Starting with a very small population of one of the predators, the orbit wanders during a long time from the preponderance of a predator to that of the other and the ultimate trend is a spiral path curling towards the periodic orbit. This behavior is easily explained in terms of stability and instability of some partial elements, whereas it seems very much alike a random behavior.

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