Time-delay feedback control in oscillatory reaction-diffusion systems

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Reaction-diffusion systems are a paradigm for the study of nonlinear dynamical processes that are taking place in a spatially extended medium. Examples can be found in many natural systems, ranging from physics and chemistry to biology, and also in areas as different as finances and cultural anthropology. Common aspects of these systems are that they show coherent temporal and spatio-temporal behaviour, reflected by travelling wave solutions, uniform oscillations, spatially-periodic but temporally constant patterns (like the famous Turing patterns), or localised patterns like spots. Over the years, the focus of the research of these systems has moved towards the (self-)engineering of patterns and systems, notably to the inclusion of feedback loops, designed for stabilising spatio-temporal chaos. These feedback loops can be a consequence of the system dynamics itself (are intrinsic), and are often operating with a time delay since assuming an instantaneous feedback is unrealistic in many cases.

Using the complex Ginzburg-Landau equation as basic model, we review some of the pattern formation occurring in oscillatory reaction-diffusion systems and then discuss in more detail the scenario when a time-delay feedback is added. The feedback contains in its most general form spatially-averaged (global) terms and local terms and the observed patterns strongly depend on the strength and nature of the feedback, as well as on the delay time. In particular, we discuss the following solutions as a function of the local feedback strength: (a) uniform oscillations, (b) birhythmicity, (c) the uniform stationary state (the stabilisation of this solution is also called amplitude death), (d) standing waves, (e) travelling waves. We use standard analytical methods like linear stability analysis and also compare to numeric simulations. This work was initially motivated by a paper by Alfonso Casal and Ildefonso Díaz [1] which also served as starting point for our fruitful collaboration and friendship that I am very grateful for.

References

[1] A. C. Casal and J. I. Díaz, Math. Models Meth. Appl. Sci. 16 (2006), 1.

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