Space-Time Analyticity of Solutions to Linear Parabolic Systems and Mathematical Finance

Peter Takáč [1] [2]

We begin by a brief presentation of a well-known mathematical model for European option pricing in a market with stochastic volatility. European options are used for *market completion*. We explain the connection between a complete market and the analyticity of the weak solution to a general, strongly parabolic linear Cauchy problem of second order in $\mathbb{R}^N \times (0,T)$ with analytic coefficients (in space and time variables). The analytic smoothing property is expressed in terms of holomorphic continuation of global (weak) L^2-type solutions to the system. Given $0 < T' < T \leq \infty$, we sketch a proof that any L^2-type solution $u\colon \mathbb{R}^N \times (0,T) \to \mathbb{R}^M$ possesses a bounded holomorphic continuation $u(x+\mathrm{i}y, \sigma+\mathrm{i}\tau)$ into a complex domain in $\mathbb{C}^N \times \mathbb{C}$ defined by $(x,\sigma) \in \mathbb{R}^N \times (T',T)$, $|y| < A'$ and $|\tau| < B'$, where $A', B' > 0$ are constants depending upon T'. The proof uses the extension of a solution to an L^2-type solution in a domain in $\mathbb{C}^N \times \mathbb{C}$, such that this extension satisfies the Cauchy-Riemann equations. The holomorphic extension is thus obtained in a Hardy space H^2.

Keywords: Space-time analyticity, parabolic PDE; holomorphic continuation, Hardy space; market completeness, European option

2000 Mathematics Subject Classification: Primary 35B65, 35K10; Secondary 91B28, 91B70;

[1] Institut für Mathematik, Universität Rostock, Ulmenstraße 69, Haus 3, D-18051 Rostock, Germany,
e-mail: peter.takac@uni-rostock.de
Web: http://www.math.uni-rostock.de/forschung/AngAnalysis

[2] Research supported in part by the D.F.G. (German Research Foundation).