FAST SPLITTING ALGORITHMS FOR CONVEX OPTIMIZATION. BEYOND NESTEROV COMPLEXITY BOUND $O(1/k^2)$.

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Abstract

Large scale optimization problems naturally appear in the modeling of many scientific and engineering situations. To meet the challenges posed by these issues, in recent years, considerable efforts have been devoted to the study of first-order splitting algorithms. The *forward-backward algorithm*, (also called the *proximal-gradient* algorithm) which is one of the most important, is a powerful tool for solving optimization problems with a *additively separable* and *smooth* plus *nonsmooth* structure. In the convex setting, a simple but ingenious acceleration scheme developed by Nesterov, and Beck-Teboulle improves the theoretical rate of convergence for the function values, in the worst case, from the standard $\mathcal{O}(k^{-1})$ down to $\mathcal{O}(k^{-2})$. In this lecture, we show that the rate of convergence of a slight variant of this accelerated forward-backward method, which produces *convergent* sequences, is actually $o(k^{-2})$, rather than $\mathcal{O}(k^{-2})$. Our arguments are based on the connection between this algorithm and a second-order differential inclusion with vanishing damping, recently introduced by Su, Boyd and Candès. The key point is the introduction of energy-like Lyapunov functions, with adapted scaling. Linking algorithms with dynamical systems provide connections between different areas, and a valuable guide for the proofs. Finally, we consider the hierarchical multi-objective problem which consists in finding by rapid methods the solution with minimum norm of a convex minimization problem. To this end, we introduce into the dynamics and algorithms a Tikhonov regularization term with vanishing coefficient. Applications are given in sparse optimization for signal/imaging processing, and inverse problems.

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Beyond Hardy-Sobolev inequality: linear diffusion equations with singular absorption potentials and/or unbounded convective flows

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Abstract. Many different applications lead to the study of linear diffusion equations of the type

$$-\Delta u + \mathbf{v}(x) \cdot \nabla u + V(x)u = f(x), \text{ in } \Omega, \tag{1}$$

where $V(x)$ is a very singular potential near the boundary of the domain $\partial\Omega$ and/or the convection coefficient $\mathbf{v}(x)$ is unbounded. This is the case, for instance, in studies on the *vorticity equation in fluid mechanics, Schrödinger equation with localizing singular potentials*, *linearization processes applied to singular and/or degenerate nonlinear equations*, *shape optimization in Chemical Engineering*, etc.

After a famous paper by H. Brezis and X. Cabré, in 1998, many authors devoted great attention to show that certain simple singular linear and nonlinear PDE's do not admit any solution u if the potential $V(x)$ grows as $Kd(x, \partial\Omega)^{-2}$. Probably this is one of the reasons why the mathematical treatment of so relevant cases is lacking of some detailed study up our days.

The main goal of this lecture is to report on a recent joint work of the author with D. Gómez-Castro, J.M. Rakotoson and R. Temam in which we prove the existence, uniqueness and give some additional regularity of solutions to equation (1) once that we introduce a correct notion of solution (which requires to work with suitable weighted spaces). To fix ideas, we shall consider mainly the case of Dirichlet boundary conditions $u = 0$ on $\partial\Omega$, but our weighted space approach can be also adapted to the case of Neumann boundary conditions. Here Ω is an open bounded smooth set of $\mathbb{R}^N$, $N \geqslant 2$. The external forcing term $f(x)$ will be assumed in $L^1(\Omega; \delta)$, where the weight in this space is given by $\delta(x) = d(x, \partial\Omega)$, and our key assumption is that the potential V is only measurable and bounded from below. With respect to the convective flow vector $\mathbf{v}(x)$ we only assume that

$$\begin{cases} \mathbf{v} \in L^N(\Omega)^N, \ \operatorname{div} \mathbf{v} = 0 & \text{in} \quad \mathcal{D}'(\Omega), \\ \mathbf{v} \cdot \mathbf{n} = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where $\mathbf{n}$ denotes the unit exterior normal vector to $\partial\Omega$ (notice that this is considerably weaker than the usual regularity obtained from the stardard $H^1(\Omega)^N$ treatment of the Navier-Stokes equation).

As a matter of fact, what is quite remarkable, we shall prove the uniqueness (and the existence) of a solution when the potential $V(x)$ is only bounded from below by $K\,\delta(x)^{-r}$, near the boundary, for some $r > 2$ and $K > 0$. Clearly, this theory goes beyond many other technics founded on the application of the Hardy-Sobolev inequality, in contrast with so many research works dealing with singular elliptic equations developed in the last twenty years.

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SWEEPING PROCESS APPROACH TO QUASISTATIC FRICTIONAL CONTACT PROBLEMS.

Samir Adly

Abstract

In this talks, we reformulate the quasistatic frictional contact problem for linear elastic materials as a new variant of the Moreau's sweeping process with velocity constraint. It is well-known that the variational formulation of this mechanical problem leads to an evolution variational inequality. The sweeping process approach offers much greater flexibility to show the well-posedness (existence and uniqueness) in a general framework. An adapted version of the Moreau's catching-up algorithm to this problem is the essential key. The link between the sweeping process and the quasistatic frictional contact problems is possible thanks to some standard tools from convex analysis and are new in the literature.

A BI-PROJECTION METHOD FOR BINGHAM TYPE FLOWS

Thierry Dubois

Abstract : In this talk, a new numerical scheme to compute isothermal and unsteady flow of an incompressible viscoplastic Bingham medium will be presented. The main difficulty, for both theoretical and numerical approaches, is due to the nondifferentiability of the plastic part of the stress tensor in regions where the rate-of-strain tensor vanishes. This is handled by reformulating the definition of the plastic stress tensor in terms of a projection. A new time scheme, based on the classical incremental projection method for the Newtonian Navier-Stokes equations, is proposed. The plastic tensor is treated implicitly in the first sub-step of the projection scheme and is computed by using a fixed point procedure. A pseudo-time relaxation is added into the Bingham projection whose effect is to ensure a geometric convergence of the fixed point algorithm. Stability and error analyses of the numerical scheme will be shown. Numerical results, obtained on the well-known two-dimensional lid-driven cavity test case, will be detailed for Reynolds number up to 10 000 and Bingham number up to 100.

AN OVERVIEW OF GRAND AND SMALL LEBESGUE SPACES,

Fiorenza Alberto, Universita di Napoli, Italie

Abstract:

A survey about grand and small Lebesgue spaces will be presented, along with their Sobolev analogues. After a short background, it will be shown the original motivation for the introduction of these spaces, the definition and the main properties, among which a description in terms of Interpolation theory. Among the applications, the role of these spaces in Sobolev-type embedding results and in PDEs theory (either existence and uniqueness results, either regularity results) will be highlighted.

Numerical simulations for stochastic partial differential equations with adaptive multilevel splitting.

Ludovic Goudenège, CNRS, CMLA

Abstract
Even with an efficient deterministic numerical method, the difficulties in numerical simulations for stochastic partial differential equations are obviously the treatment of the noise.
Actually the question is : "What is the quantity of interest ?". It is almost impossible to obtain all the behaviour of the solutions (i.e. the whole density). And even for studying a specific behaviour, we need a heavy Monte-Carlo method (unreachable in reasonable time in practice). Rarer is the event of interest, worse is the problem. In this talk I will present a numerical method based on splitting operator in time, and an adaptive multilevel splitting algorithm to simulate rare events for stochastic partial differential equations.

GLOBAL EXISTENCE FOR SYSTEMS DESCRIBING MULTICOMPONENT REACTIVE FLOW,

Martine Marion, de L'Ecole Centrale de Lyon.

Abstract
We consider combustion problems in the presence of complex chemistry and nonlinear diffusion laws for the chemical species. The nonlinear diffusion coefficients are obtained by resolution of the so-called Stefan-Maxwell equations. We prove the existence of weak solutions for the corresponding system of equations which involves coupling between the incompressible Navier-Stokes and equations for temperature and species concentrations.

NEW PERSPECTIVES IN SOLVING CARDIAC ELECTROPHYSIOLOGICAL ACTIVITY RECONSTRUCTION

Abdeljalil Nachaoui, Universit de Nantes

ELECTROCARDIOGRAPHY (ECG) investigates the relationship between the electrical activity of the heart and its induced voltages measured on the torso surface. This relationship can be characterized mathematically as an inverse problem where the goal is to noninvasively estimate cardiac electrical activity from voltage distributions measured on the body surface. In order to solve this problem we suggest a new approach based on domain decomposition technique. We approximate our approach by a Finite Element Method. Numerical experiments with 2D domains highlight the efficiency of the proposed methods as well as their robustness in the model context.

RECIPROCAL RICCATI TRANSFORMATION AS A NEW METHOD IN ODES FOR PROVING THE NON-MONOTONIC BEHAVIOUR OF PARTICLE DENSITY IN BEC

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Abstract
The so called reciprocal Riccati transformation is introduced for the purpose of obtaining the non-monotonic solutions of the corresponding nonlinear ODEs. Next, by using usual tricks, it is repeated to the solitary wave (ground state) solutions of the nonlinear Schrodinger equations with non-homogeneous coefficients. Finally, it is applied in some concrete problems in BEC in order to theoretically explain some numerical simulations which show that the particle density can be non-monotonic.

PLEASANT FUNCTIONS AND SETS

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Whereas wild functions are known since the work of Weierstrass, it is of interest to shed some light on functions that are of pleasant use, even if they are nonsmooth. Desirable characters are: coincidence of some of the classical subdifferentials for such functions, links with convexity, possibility of approximation by smooth functions. Characterizations and stability properties are part of our program.

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THE SCHRÖDINGER OPERATOR WITH SINGULAR POTENTIALS

Augusto C. Ponce

The Schrödinger operator $-\Delta + V$ in $\mathbb{R}^N$ has been extensively studied for potentials in L^∞ and even L^p with any exponent $p > N/2$. Kato's inequality in the 1970s was a major breakthrough in spectral problems by allowing one to consider potentials V that are merely L^1. We present new counterparts of the strong maximum principle and Hopf's boundary lemma for $-\Delta + V$ on domains when V is singular.

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Sharp-diffuse interfaces model for a seawater intrusion problem

Carole Rosier

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Abstract

We derive a new model describing the displacement of saltwater front in coastal aquifers. It combine the efficiency of the sharp interface approach with the physical realism of the diffuse interface one. The three-dimensional problem is then reduced to a two-dimensional model involving a strongly coupled system of pdes of parabolic type describing the evolution of the depths of the two free surfaces.
From a theoretical point of view, an advantage resulting from the addition of diffuse area compared to the sharp interface approximation is that the system now has a parabolic structure, so we can demonstrate a more efficient and logical maximum principle from the point of view of physics than in the classical case of sharp interface approximation. Moreover, we also establish the uniqueness of the solution thanks to the parabolic structure of the system which allows to get more regularity for the solution.

PROBLÈMES DE RELÈVEMENTS DANS LES ESPACES DE BESOV

EMMANUEL RUSS

Soient $s > 0$ et $p \in (1, +\infty)$. Soit $\Omega \subset \mathbb{R}^n$ un domaine borné régulier et simplement connexe. Si $u : \Omega \to \mathbb{S}^1$ appartient à l'espace de Sobolev fractionnaire $W^{s,p}$, peut-on écrire $u = e^{i\varphi}$ avec $\varphi \in W^{s,p}(\mathbb{R})$? La réponse à cette question, due à Bourgain, Brezis et Mironescu, est connue pour toutes les valeurs de s et de p. Nous examinerons le problème analogue dans le cadre des espaces de Besov $B^s_{p,q}$, qui fait apparaître des phénomènes inattendus. Cet exposé s'appuie sur des travaux avec P. Mironescu et Y. Sire.

LIFTING PROBLEMS IN BESOV SPACES

EMMANUEL RUSS

Let $s > 0$ and $p \in (1, +\infty)$. Let $\Omega \subset \mathbb{R}^n$ be a bounded simply connected and smooth domain. If $u : \Omega \to \mathbb{S}^1$ belongs to the fractional $W^{s,p}(\Omega)$ Sobolev space, does there exist a function $\varphi \in W^{s,p}(\Omega, \mathbb{R})$ such that $u = e^{i\varphi}$? The answer, due to Bourgain, Brezis and Mironescu, is known for the full range of s and p. We will deal with the analogous issue in the context of Besov $B^s_{p,q}$ spaces, where several unexpected phenomena occur. This talk relies on joint works with P. Mironescu and Y. Sire.

Perturbation of error bounds

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In the current presentation, I intend to extend the developments in Kruger, Ngai & Théra, SIAM J. Optim. **20**(6), 3280–3296 (2010) and, more precisely, to characterize, in the Banach space setting, the stability of the local and global error bound property of inequalities determined by proper lower semicontinuous under data perturbations. I will propose new concepts of (arbitrary, convex and linear) perturbations of the given function defining the system under consideration, which turn out to be a useful tool in our analysis. The characterizations of error bounds for families of perturbations can be interpreted as estimates of the 'radius of error bounds'. The definitions and characterizations are illustrated by examples. This presentation summarizes recent joint works with Huynh Van Ngai, Marco López Cerda and A. Kruger.