Using general material constitutive relations in numerical methods for structural models: algorithms and variational basis

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ABSTRACT

Linear and nonlinear models of bars, beams, plates and shells have governing equations whose complete form can only be obtained when their material is linear and elastic. Many interesting situations, however, demand the use of nonlinear and inelastic constitutive laws for the material. Only then phenomena such as plasticity, damage, viscoelasticity, etc. can be incorporated into the model.

Numerical methods for structural models, both linear and nonlinear, can employ arbitrary constitutive laws for the material only by integrating the section response and iteratively solving the strain-stress relations under the kinematic constraints of the model. For this, a Newton-Raphson or similar scheme must be employed at every quadrature point of the cross section [1, 2].

Iterative procedures as the ones previously indicated are blind to the energetic underpinnings of equilibrium problems, be them of continua or structures. The first result that we will present is the identification of an energy function that serves as variational basis of the section response, for all types of structural models. This function, when appropriately integrated, can be used in defining a new minimizing principle that governs the equilibria of structures, including those employing nonlinear, inelastic three-dimensional constitutive laws for their material response.

This result has several consequences: first, the numerical solution of inelastic structures will be obtainable without the computation of the tangent stiffness, merely by using simple descent-type methods. Second, the obtained energy can serve as error indicator for the most general and complex structural problems. Numerical examples of these two aspects will be shown.

References

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[2] Klinkel, S., & Govindjee, S. (2002). Using finite strain 3D-material models in beam and shell elements. Engineering Computations, 19(8), 902–921.