

---

# Curves and Surfaces

Departamento de Matemática Aplicada  
ETS Arquitectura  
Universidad Politécnica de Madrid

---

# Curves and Surfaces

---

**'Curves and Surfaces' is taught in the first (and second) semester of the second year of the degree in Architecture (UPM).**

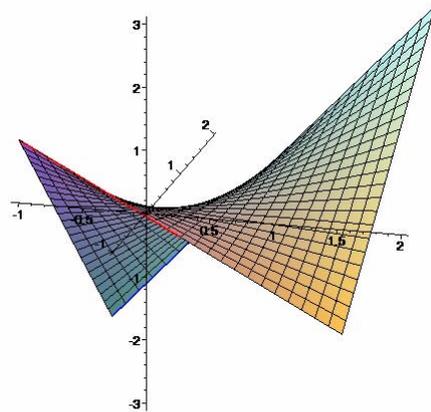
**This course covers the concepts and mathematical tools to address issues of 'measure' and 'shape' in curves and surfaces.**

**To arise and solve geometric problems involving planar curves and twisted curves, surfaces of revolution, ruled surfaces, ...**

# Curves and Surfaces

---

differential geometry of curves and surfaces

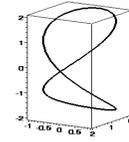


Connections  
between  
and  
Architecture  
will be  
established  
in the classes.



# 1. Curves

---



1.1. Parametrizations. Parametrization of conics. Implicit equations of a curve. Planar curves: implicit, explicit equations and equations in polar coordinates.

1.2. Length of a regular curve. Line integral of a scalar-valued function. Arc length parameter (or natural parameter).

1.3. Frenet frame. Planes and lines associated with Frenet frame. Projections of a curve on planes.

1.4. Curvature and torsion. Radius of curvature. Osculating circle. Frenet Serret formulas.

1.5. Significant curves: helices.

# 2. Surfaces

---

- 2.1. Parametrizations. Parametrizations of quadric surfaces. Implicit and explicit equations. Parametrizations by rotation, translation, etc. Surfaces of revolution.
- 2.2. Tangent plane and Normal line at a regular point. Parametric curves.
- 2.3. First fundamental form. Measure. Surface integral of scalar function.
- 2.4. Second fundamental form. Curvatures of the curves of a surface. Normal curvature.
- 2.5. Asymptotic directions. Asymptotic curves.
- 2.6. Classification of regular points of a surface: elliptic, hyperbolic and parabolic points. Umbilic points and planar points.
- 2.7. Principal curvatures. Gaussian curvature. Mean curvature. Principal directions. Lines of Curvature. Geodesics. Euler formula.

# Ruled Surfaces

---

2.8. Ruled Surfaces. Directrix and rulings. Parametrization of a ruled surface.

2.9. Measure and shape of a ruled surface: first and second fundamental forms. distribution parameter. Properties of the ruled surfaces.

2.10. Classification of ruled surfaces: developable surfaces and non-developable surfaces. Properties. Edge of regression. Striction curve.

