Circles on surfaces

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Abstract

The sphere in 3-space has an infinite number of circles through any closed point. The torus
has 4 circles through any closed point. Two of these circles are known as Villarceau circles
([0]). We define a “celestial” to be a real surface with at least 2 real circles through a generic
closed point. Equivalently, a celestial is a surface with at least 2 families of real circles.

In 1980 Blum [1] conjectured that a real surface has either at most 6 families of circles
or an infinite number. For compact surfaces this conjecture has been proven by Takeuchi [2]
least 2 families of conics. This result together with Moebius geometry led to a classification
of celestials in 3-space [4].

In 2012 Pottmann et al. [5] conjectured that a surface in 3-space with exactly 3 circles
through a closed point is a Darboux Cyclide. We confirm this conjecture as a corollary from
our classification in [4].

We recall that a translation is an isometry where every point moves with the same distance.
In this talk we consider celestials in 3-space that are obtained from translating a circle along
a circle, in either Euclidean or elliptic space. This is a natural extension of classical work by
William Kingdon Clifford and Felix Klein on the Clifford torus.

Krasauskas, Pottmann and Skopenkov conjectured, that celestials in 3-space of Moebius
degree 8 are Moebius equivalent to an Euclidean or Elliptic translational celestial. This conjec-
ture is true if its Moebius model has a family of great circles ([6]). Moreover, its real singular
locus consist of a great circle. As a corollary we obtain a classically flavored theorem in elliptic
geometry: if we translate a line along a circle but not along a line then exactly 2 translated
lines will coincide ([6]).

Keywords
families of circles, cyclides, Moebius geometry, elliptic geometry, weak Del Pezzo surfaces,
translational surfaces

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