```
> restart:with(linalg):
```

Ejercicio 1

La matriz del sistema de EDOs es

```
> M:=matrix(3,3,[[4,-2,0],[1,4,1],[-3,0,1]]);
```

$$M := \begin{bmatrix} 4 & -2 & 0 \\ 1 & 4 & 1 \\ -3 & 0 & 1 \end{bmatrix} \tag{1.1}$$

Vector de términos independientes:

```
> [0,t,2];
```

$$[0, t, 2] \tag{1.2}$$

Soluciones del sistema homogéneo asociado:

```
> eigenvectors(M);
```

$$\left[2, 1, \left\{\left[\begin{array}{ccc} 1 & 1 & -3 \end{array}\right]\right\}\right], \left[4, 1, \left\{\left[\begin{array}{ccc} -1 & 0 & 1 \end{array}\right]\right\}\right], \left[3, 1, \left\{\left[\begin{array}{ccc} 2 & 1 & -3 \end{array}\right]\right\}\right] \tag{1.3}$$

Sistema fundamental de soluciones

```
> [2,1,-3]*exp(3*t);
```

$$[2, 1, -3]\, e^{3t} \tag{1.4}$$

```
> [1,1,-3]*exp(2*t);
```

$$[1, 1, -3]\, e^{2t} \tag{1.5}$$

```
> [-1,0,1]*exp(4*t);
```

$$[-1, 0, 1]\, e^{4t} \tag{1.6}$$

Matriz fundamental del sistema

```
> Phi(t):=concat([2,1,-3]*exp(3*t),[1,1,-3]*exp(2*t),[-1,0,1]*
  exp(4*t));
```

$$\Phi(t) := \begin{bmatrix} 2\, e^{3t} & e^{2t} & -e^{4t} \\ e^{3t} & e^{2t} & 0 \\ -3\, e^{3t} & -3\, e^{2t} & e^{4t} \end{bmatrix} \tag{1.7}$$

Cálculo de la solución particular del sistema:

```
> v:=evalm(inverse(Phi(t))&*[0,t,2]);
```

$$v := \left[\begin{array}{ccc} \frac{2t}{e^{3t}} + \frac{2}{e^{3t}} & -\frac{t}{e^{2t}} - \frac{2}{e^{2t}} & \frac{3t}{e^{4t}} + \frac{2}{e^{4t}} \end{array}\right] \tag{1.8}$$

```
> Int(v[1],t)=int(v[1],t);
  Int(v[2],t)=int(v[2],t);
  Int(v[3],t)=int(v[3],t);
```

$$\int \left(\frac{2t}{e^{3t}} + \frac{2}{e^{3t}}\right) dt = -\frac{2}{9}\,\frac{4+3t}{e^{3t}}$$

$$\int \left(-\frac{t}{e^{2t}} - \frac{2}{e^{2t}}\right) dt = \frac{1}{4}\,\frac{5+2t}{e^{2t}}$$

$$\int \left(\frac{3t}{e^{4t}} + \frac{2}{e^{4t}}\right) dt = -\frac{1}{16}\,\frac{11+12t}{e^{4t}} \tag{1.9}$$

Solución particular del sistema:

```
> x:=simplify(evalm(Phi(t)&*[int(v[1],t),int(v[2],t),int(v[3],t)]));
```

$$x := \left[\frac{23}{144} - \frac{1}{12}\,t \quad \frac{13}{36} - \frac{1}{6}\,t \quad -\frac{85}{48} - \frac{1}{4}\,t \right] \tag{1.10}$$

Comprobación:

```
> evalm(diff([x[1],x[2],x[3]],t)-M&*[x[1],x[2],x[3]]-[0,t,2]);
```

$$\left[0 \quad 0 \quad 0 \right] \tag{1.11}$$

Ejercicio 2

```
> restart:with(linalg):
```

La matriz de coeficientes del sistema es:

```
> M:=matrix(3,3,[[1,-3,-3],[-1,-1,-1],[1,5,5]]);
```

$$M := \begin{bmatrix} 1 & -3 & -3 \\ -1 & -1 & -1 \\ 1 & 5 & 5 \end{bmatrix} \tag{2.1}$$

El vector de términos independientes es:

```
> [4,t,1];
```

$$[4, t, 1] \tag{2.2}$$

```
> eigenvectors(M);
```

$$\left[0, 1, \left\{\left[0 \quad -1 \quad 1 \right]\right\}\right], \left[1, 1, \left\{\left[1 \quad -1 \quad 1 \right]\right\}\right], \left[4, 1, \left\{\left[-1 \quad 0 \quad 1 \right]\right\}\right] \tag{2.3}$$

Sistema fundamental de soluciones:

```
> [-1,0,1]*exp(4*t);
```

$$[-1, 0, 1]\, e^{4t} \tag{2.4}$$

```
> [0,-1,1]*exp(0);
```

$$[0, -1, 1] \tag{2.5}$$

```
> [1,-1,1]*exp(t);
```

$$[1, -1, 1]\, e^{t} \tag{2.6}$$

Matriz fundamental del sistema:

```
> Phi(t):=concat([-1,0,1]*exp(4*t),[0,-1,1]*exp(0),[1,-1,1]*exp(t));
```

$$\Phi(t) := \begin{bmatrix} -e^{4t} & 0 & e^{t} \\ 0 & -1 & -e^{t} \\ e^{4t} & 1 & e^{t} \end{bmatrix} \tag{2.7}$$

Cálculo de la solución particular del sistema:

```
> v:=evalm(inverse(Phi(t))&*[4,t,1]);
```

$$v := \left[\frac{t}{e^{4t}} + \frac{1}{e^{4t}} \quad -5 - 2\,t \quad \frac{5}{e^{t}} + \frac{t}{e^{t}} \right] \tag{2.8}$$

```
> Int(v[1],t)=int(v[1],t);
  Int(v[2],t)=int(v[2],t);
  Int(v[3],t)=int(v[3],t);
```

$$\int\left(\frac{t}{e^{4t}}+\frac{1}{e^{4t}}\right)dt=-\frac{1}{16}\frac{5+4t}{e^{4t}}$$
$$\int(-5-2t)\,dt=-5t-t^2$$
$$\int\left(\frac{5}{e^{t}}+\frac{t}{e^{t}}\right)dt=-\frac{6+t}{e^{t}} \tag{2.9}$$

Solución particular del sistema:

```
> x:=simplify(evalm(Phi(t)&*[int(v[1],t),int(v[2],t),int(v[3],
t)]));
```

$$x:=\left[\begin{array}{ccc}-\frac{91}{16}-\frac{3}{4}t & 6+6t+t^2 & -\frac{101}{16}-\frac{25}{4}t-t^2\end{array}\right] \tag{2.10}$$

Comprobación:

```
> evalm(diff([x[1],x[2],x[3]],t)-M&*[x[1],x[2],x[3]]-[4,t,1]);
```

$$\left[\begin{array}{ccc}0 & 0 & 0\end{array}\right] \tag{2.11}$$

Ejercico 3

```
> restart:with(linalg):
> M:=matrix(3,3,[[1,-3,-3],[0,1,0],[0,3,4]]);
```

$$M:=\left[\begin{array}{ccc}1 & -3 & -3\\ 0 & 1 & 0\\ 0 & 3 & 4\end{array}\right] \tag{3.1}$$

Vector de términos independientes:

```
> [5,t^2,t-1];
```

$$[5, t^2, t-1] \tag{3.2}$$

```
> eigenvectors(M);
```

$$\left[1, 2, \left\{\left[\begin{array}{ccc}0 & -1 & 1\end{array}\right], \left[\begin{array}{ccc}1 & 0 & 0\end{array}\right]\right\}\right], \left[4, 1, \left\{\left[\begin{array}{ccc}-1 & 0 & 1\end{array}\right]\right\}\right] \tag{3.3}$$

Sistema fundamental de soluciones:

```
> [0,-1,1]*exp(t);
```

$$[0, -1, 1]\,e^{t} \tag{3.4}$$

```
> [1,0,0]*exp(t);
```

$$[1, 0, 0]\,e^{t} \tag{3.5}$$

```
> [-1,0,1]*exp(4*t);
```

$$[-1, 0, 1]\,e^{4t} \tag{3.6}$$

Matriz fundamental del sistema:

```
> Phi(t):=concat([0,-1,1]*exp(t),[1,0,0]*exp(t),[-1,0,1]*exp(4*
t));
```

$$\Phi(t):=\left[\begin{array}{ccc}0 & e^{t} & -e^{4t}\\ -e^{t} & 0 & 0\\ e^{t} & 0 & e^{4t}\end{array}\right] \tag{3.7}$$

Cálculo de la solución particular del sistema:

```
> v:=evalm(inverse(Phi(t))&*[5,t^2,t-1]);
```

$$v := \left[\begin{array}{ccc} -\frac{t^2}{e^t} & \frac{5}{e^t}+\frac{t^2}{e^t}+\frac{t-1}{e^t} & \frac{t^2}{e^{4t}}+\frac{t-1}{e^{4t}} \end{array}\right] \quad (3.8)$$

```
> Int(v[1],t)=int(v[1],t);
  Int(v[2],t)=int(v[2],t);
  Int(v[3],t)=int(v[3],t);
```

$$\int\left(-\frac{t^2}{e^t}\right)dt = \frac{2+2t+t^2}{e^t}$$

$$\int\left(\frac{5}{e^t}+\frac{t^2}{e^t}+\frac{t-1}{e^t}\right)dt = -\frac{7+3t+t^2}{e^t}$$

$$\int\left(\frac{t^2}{e^{4t}}+\frac{t-1}{e^{4t}}\right)dt = -\frac{1}{32}\,\frac{-5+12t+8t^2}{e^{4t}} \quad (3.9)$$

Solución particular del sistema:

```
> x:=simplify(evalm(Phi(t)&*[int(v[1],t),int(v[2],t),int(v[3],
  t)]));
```

$$x := \left[\begin{array}{ccc} -\frac{229}{32}-\frac{21}{8}t-\frac{3}{4}t^2 & -2-2t-t^2 & \frac{69}{32}+\frac{13}{8}t+\frac{3}{4}t^2 \end{array}\right] \quad (3.10)$$

Comprobación:

```
> evalm(diff([x[1],x[2],x[3]],t)-M&*[x[1],x[2],x[3]]-[5,t^2,
  t-1]);
```

$$\left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right] \quad (3.11)$$

Ejercicio 4

La matriz del sistema de EDOs es

```
> M:=matrix(3,3,[[-2,-1,0],[0,1,0],[3,1,1]]);
```

$$M := \left[\begin{array}{ccc} -2 & -1 & 0 \\ 0 & 1 & 0 \\ 3 & 1 & 1 \end{array}\right] \quad (4.1)$$

Vector de términos independientes:

```
> [0,t,2];
```

$$[0, t, 2] \quad (4.2)$$

Soluciones del sistema homogéneo asociado:

```
> eigenvectors(M);
```

$$\left[-2, 1, \left\{\left[\begin{array}{ccc} -1 & 0 & 1 \end{array}\right]\right\}\right], \left[1, 2, \left\{\left[\begin{array}{ccc} 0 & 0 & 1 \end{array}\right], \left[\begin{array}{ccc} 1 & -3 & 0 \end{array}\right]\right\}\right] \quad (4.3)$$

Sistema fundamental de soluciones

```
> [0,0,1]*exp(t);
```

$$[0, 0, 1]\, e^t \quad (4.4)$$

```
> [1,-3,0]*exp(t);
```

$$[1, -3, 0]\, e^t \quad (4.5)$$

```
> [-1,0,1]*exp(-2*t);
```

$$[-1, 0, 1]\,e^{-2t} \tag{4.6}$$

Matriz fundamental del sistema

```
> Phi(t):=concat([0,0,1]*exp(t),[1,-3,0]*exp(t),[-1,0,1]*exp
  (-2*t));
```

$$\Phi(t) := \begin{bmatrix} 0 & e^{t} & -e^{-2t} \\ 0 & -3\,e^{t} & 0 \\ e^{t} & 0 & e^{-2t} \end{bmatrix} \tag{4.7}$$

```
> evalm(Phi(t)&*[C1,C2,C3]);
```

$$\begin{bmatrix} e^{t}\,C2 - e^{-2t}\,C3 & -3\,e^{t}\,C2 & e^{t}\,C1 + e^{-2t}\,C3 \end{bmatrix} \tag{4.8}$$

Cálculo de la solución particular del sistema:

```
> v:=evalm(inverse(Phi(t))&*[0,t,2]);
```

$$v := \begin{bmatrix} \frac{1}{3}\,\frac{t}{e^{t}} + \frac{2}{e^{t}} & -\frac{1}{3}\,\frac{t}{e^{t}} & -\frac{1}{3}\,\frac{t}{e^{-2t}} \end{bmatrix} \tag{4.9}$$

```
> Int(v[1],t)=int(v[1],t);
  Int(v[2],t)=int(v[2],t);
  Int(v[3],t)=int(v[3],t);
```

$$\int \left(\frac{1}{3}\,\frac{t}{e^{t}} + \frac{2}{e^{t}} \right) \mathrm{d}t = -\frac{1}{3}\,\frac{7+t}{e^{t}}$$

$$\int \left(-\frac{1}{3}\,\frac{t}{e^{t}} \right) \mathrm{d}t = \frac{1}{3}\,\frac{1+t}{e^{t}}$$

$$\int \left(-\frac{1}{3}\,\frac{t}{e^{-2t}} \right) \mathrm{d}t = -\frac{1}{12}\,\frac{-1+2t}{e^{-2t}} \tag{4.10}$$

Solución particular del sistema:

```
> x:=simplify(evalm(Phi(t)&*[int(v[1],t),int(v[2],t),int(v[3],
  t)]));
```

$$x := \begin{bmatrix} \frac{1}{4} + \frac{1}{2}\,t & -1 - t & -\frac{9}{4} - \frac{1}{2}\,t \end{bmatrix} \tag{4.11}$$

Comprobación:

```
> evalm(diff([x[1],x[2],x[3]],t)-M&*[x[1],x[2],x[3]]-[0,t,2]);
```

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{4.12}$$

```
> 
```