Ejercicio 1

Consideramos la función $f(x, y) = xy^2 + 3$. *Se pide* :

```
> restart :
> with(plots) : with(linalg) :
```

Gráfica de *f*

```
> f := (x, y) → x·y^2 + 3;
```

$$f := (x, y) \to x\,y^2 + 3 \qquad \textbf{(1.1.1)}$$

```
> G := plot3d( f(x, y), x = -2 ..2, y = -2 ..2, axes = normal) :
```

Curva C imagen de la recta del plano z=0 que contiene al punto (0,0, 0) y tiene vector director v=(1,1,0)

```
> P := [0, 0, 0];
  v := [1, 1, 0];
```

$$P := [0, 0, 0]$$
$$v := [1, 1, 0] \qquad \textbf{(1.2.1)}$$

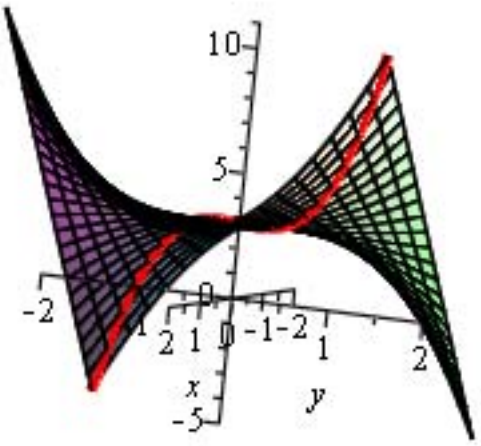
```
> P + t·v;
```

$$[0, 0, 0] + t\,[1, 1, 0] \qquad \textbf{(1.2.2)}$$

```
> r := t → [t, t, f(t, t)];
```

$$r := t \to [t, t, f(t, t)] \qquad \textbf{(1.2.3)}$$

```
> C := spacecurve(r(t), t = -2 ..2, thickness = 3, color = red) :
> display(G, C);
```



Recta tangente a la curva C en el punto (0,0,f(0,0))

```
> [0, 0, f(0, 0)];
  r(0);
```

$$[0, 0, 3]$$

$$[0, 0, 3] \quad \textbf{(1.3.1)}$$

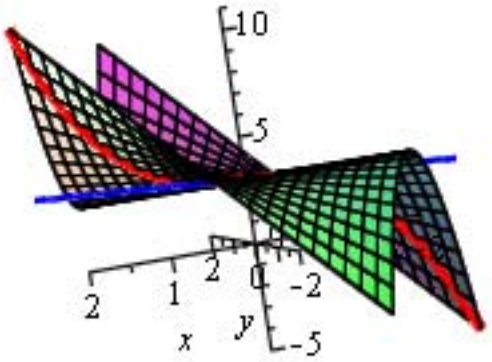
```
> diff(r(t), t);
```

$$[1, 1, 3\,t^2] \quad \textbf{(1.3.2)}$$

```
> rt := t→[0, 0, 3] + t·[1, 1, 0];
```

$$rt := t \rightarrow [0, 0, 3] + t\,[1, 1, 0] \quad \textbf{(1.3.3)}$$

```
> RectaTangente := spacecurve([t, t, 3], t = -2..2, color = blue, thickness = 2):
> display(G, C, RectaTangente);
```



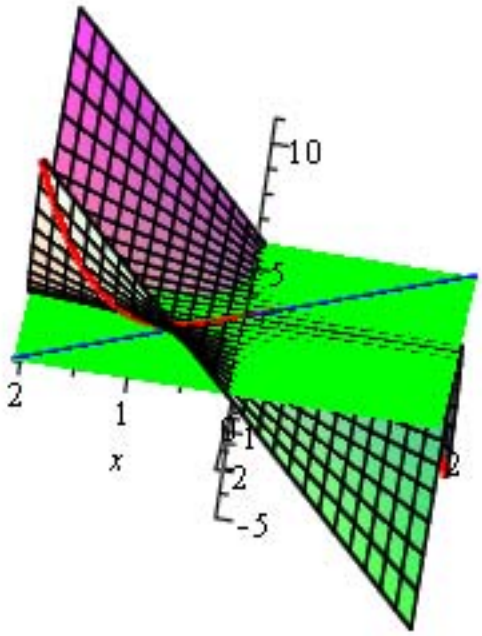
Plano tangente a la gráfica de f en el punto P

> $z = f(0,0) + \mathrm{D}[1](f)(0,0)\cdot(x-0) + \mathrm{D}[2](f)(0,0)\cdot(y-0);$

$$z = 3 \qquad \textbf{(1.4.1)}$$

> $PlanoTangente := implicitplot3d(z = 3, x = -2..2, y = -2..2, z = 2..4, color = green, style = surface):$

> $display(G, C, RectaTangente, PlanoTangente);$



Curva C1 imagen de la recta del plano z=0 que contiene al punto (1, 1,0) y tiene vector director v=(1,0,0)

```
> P1 := [1, 1, 0];
  v1 := [1, 0, 0];
```

$$P1 := [1, 1, 0]$$
$$v1 := [1, 0, 0] \quad \textbf{(1.5.1)}$$

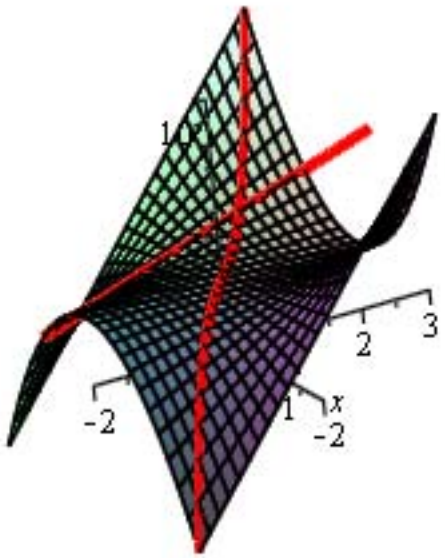
```
> P1 + t·v1;
```

$$[1, 1, 0] + t\,[1, 0, 0] \quad \textbf{(1.5.2)}$$

```
> r1 := t→[1 + t, 1, f(1 + t, 1)];
```

$$r1 := t \rightarrow [1 + t, 1, f(1 + t, 1)] \quad \textbf{(1.5.3)}$$

```
> C1 := spacecurve(r1(t), t = -3..2, thickness = 3, color = red) :
> display(G, C, C1);
```



Punto de intersección de las curvas C y C1

```
> r(t);
  r1(s);
```

$$[t,\ t,\ t^3+3]$$

$$[1+s,\ 1,\ 4+s] \qquad \textbf{(1.5.1.1)}$$

```
> solve({t = 1 + s, 1 = t, t^3 + 3 = 4 + s}, {t, s});
```

$$\{s=0,\ t=1\} \qquad \textbf{(1.5.1.2)}$$

```
> r(1);
  r1(0);
```

$$[1,\ 1,\ 4]$$

$$[1,\ 1,\ 4] \qquad \textbf{(1.5.1.3)}$$

Plano tangente a la gráfica de f en el punto Q intersección de C y C1

```
> [1, 1, f(1, 1)];
```

$$[1,\ 1,\ 4] \qquad \textbf{(1.5.2.1)}$$

```
> z = f(1, 1) + D[1](f)(1, 1)·(x − 1) + D[2](f)(1 , 1)·(y − 1);
```

(1.5.2.2)

$$z = 1 + x + 2\,y \qquad \textbf{(1.5.2.2)}$$

```
> PlanoTangente1 := implicitplot3d(z = 1 + x + 2 y, x = -2 ..2, y = -2 ..2, z = 2 ..5, color
      = green, style = surface) :
> display(G, C, C1, PlanoTangente1);
```