```
> restart:with(linalg):
```

Ejercicio 1

Ecuación característica:

```
> lambda^2-2*lambda+1=0;
```

$$\lambda^2 - 2\lambda + 1 = 0 \tag{1.1}$$

```
> solve(lambda^2-2*lambda+1,lambda);
```

$$1, 1 \tag{1.2}$$

Solución general de la EDO homogénea asociada:

```
> yh1:=x->C*exp(x)+D*x*exp(x);
```

$$yh1 := x \rightarrow C\,e^x + D\,x\,e^x \tag{1.3}$$

Comprobación.

```
> diff(yh1(x),x,x)-2*diff(yh1(x),x)+yh1(x);
```

$$0 \tag{1.4}$$

Imponemos que ax^2+bx+c sea solución de la ecuación:

```
> solve({a=3,-4*a+b=2,2*a-2*b+c=-20},{a,b,c});
```

$$\{a = 3, b = 14, c = 2\} \tag{1.5}$$

```
> y1:=x->yh1(x)+3*x^2+14*x+2;
```

$$y1 := x \rightarrow yh1(x) + 3\,x^2 + 14\,x + 2 \tag{1.6}$$

Comprobación:

```
> diff(y1(x),x,x)-2*diff(y1(x),x)+y1(x);
```

$$-20 + 2\,x + 3\,x^2 \tag{1.7}$$

Condiciones iniciales:

```
> y(0)=5;
```

$$y(0) = 5 \tag{1.8}$$

```
> D(y)(0)=3;
```

$$D(y)(0) = 3 \tag{1.9}$$

```
> solve({y1(0)=5,D(y1)(0)=3},{C,D});
```

$$\{C = 3, D = -14\} \tag{1.10}$$

```
> solve({y1(0)=8,D(y1)(0)=20},{C,D});
```

$$\{C = 6, D = 0\} \tag{1.11}$$

```
> solve({y1(0)=8,y1(1)=4},{C,D});
```

$$\left\{C = 6, D = -\frac{3\,(2\,e + 5)}{e}\right\} \tag{1.12}$$

Ejercicio 2

Ecuación característica:

```
> lambda^2+4=0;
```

$$\lambda^2 + 4 = 0 \tag{2.1}$$

```
> solve(lambda^2+4=0,lambda);
```

$$2\,I, -2\,I \tag{2.2}$$

Solución general de la EDO homogénea asociada:

```
> yh2:=x->C*cos(2*x)+D*sin(2*x);
```

(2.3)

$$yh2 := x \to C\cos(2x) + \mathrm{D}\sin(2x) \tag{2.3}$$

Comprobación:

```
> diff(yh2(x),x,x)+4*yh2(x);
```

$$0 \tag{2.4}$$

Imponemos que la siguiente función sea solución de la EDO:

```
> yp:=x->a*x*cos(2*x)+b*x*sin(2*x)+c*x^2+d*x+f;
```

$$yp := x \to a\,x\cos(2x) + b\,x\sin(2x) + c\,x^2 + d\,x + f \tag{2.5}$$

esto es,

```
> diff(yp(x),x,x)+4*yp(x)=4*cos(2*x)+8*x^2-4*x;
```

$$-4\,a\sin(2x) + 4\,b\cos(2x) + 2\,c + 4\,c\,x^2 + 4\,d\,x + 4\,f = 4\cos(2x) + 8\,x^2 - 4\,x \tag{2.6}$$

Obtenemos el siguiente sistema de ecuaciones:

```
> {-4*a=0,4*b=4,4*c=8,4*d=-4,2*c+4*f=0};
```

$$\{-4\,a = 0,\ 4\,b = 4,\ 4\,c = 8,\ 4\,d = -4,\ 2\,c + 4\,f = 0\} \tag{2.7}$$

```
> solve({-4*a=0,4*b=4,4*c=8,4*d=-4,2*c+4*f=0},{a,b,c,d,f});
```

$$\{a = 0,\ b = 1,\ c = 2,\ d = -1,\ f = -1\} \tag{2.8}$$

Solución particular de la EDO:

```
> yp2:=x->x*sin(2*x)+2*x^2-x-1;
```

$$yp2 := x \to x\sin(2x) + 2\,x^2 - x - 1 \tag{2.9}$$

Comprobación:

```
> diff(yp2(x),x,x)+4*yp2(x);
```

$$4\cos(2x) + 8\,x^2 - 4\,x \tag{2.10}$$

Solución general de la EDO:

```
> y2:=x->yh2(x)+yp2(x);
  y2(x);
```

$$y2 := x \to yh2(x) + yp2(x)$$

$$C\cos(2x) + \mathrm{D}\sin(2x) + x\sin(2x) + 2\,x^2 - x - 1 \tag{2.11}$$

Comprobación:

```
> diff(y2(x),x,x)+4*y2(x);
```

$$4\cos(2x) + 8\,x^2 - 4\,x \tag{2.12}$$

Condiciones iniciales:

```
> y2(0)=3;
  D(y2)(0)=8;
```

$$C - 1 = 3$$

$$-1 + 2\,\mathrm{D} = 8 \tag{2.13}$$

```
> solve({y2(0)=3,D(y2)(0)=8});
```

$$\left\{C = 4,\ \mathrm{D} = \frac{9}{2}\right\} \tag{2.14}$$

```
> y2_CI:=x->4*cos(2*x)+9/2*sin(2*x)+x*sin(2*x)+2*x^2-x-1;
```

$$y2\_CI := x \to 4\cos(2x) + \frac{9}{2}\sin(2x) + x\sin(2x) + 2\,x^2 - x - 1 \tag{2.15}$$

Comprobación:

```
> diff(y2_CI(x),x,x)+4*y2_CI(x);
```

$$4\cos(2x) + 8\,x^2 - 4\,x \tag{2.16}$$

Ejercico 3

Ecuación característica:

```
> lambda^4+6*lambda^3+14*lambda^2+16*lambda+8=0;
```

$$\lambda^4 + 6\lambda^3 + 14\lambda^2 + 16\lambda + 8 = 0 \tag{3.1}$$

```
> solve(lambda^4+6*lambda^3+14*lambda^2+16*lambda+8, lambda);
```

$$-1 + \mathrm{I},\ -1 - \mathrm{I},\ -2,\ -2 \tag{3.2}$$

Solución de la EDO homogénea:

```
> yh3:=x->A*exp(-x)*cos(x)+B*exp(-x)*sin(x)+C*exp(-2*x)+D*x*exp
  (-2*x);
```

$$yh3 := x \rightarrow A\,\mathrm{e}^{-x}\cos(x) + B\,\mathrm{e}^{-x}\sin(x) + C\,\mathrm{e}^{-2x} + \mathrm{D}\,x\,\mathrm{e}^{-2x} \tag{3.3}$$

y(x)=3 es solución particular de la EDO. La solución general de la EDO es:

```
> y3:=x->3+yh3(x);
```

$$y3 := x \rightarrow 3 + yh3(x) \tag{3.4}$$

Comprobación:

```
> simplify(diff(y3(x),x,x,x,x)+6*diff(y3(x),x,x,x)+14*diff(y3
  (x),x,x)+16*diff(y3(x),x)+8*ys3(x));
```

$$-8\,B\,\mathrm{e}^{-x}\sin(x) - 8\,A\,\mathrm{e}^{-x}\cos(x) - 8\,\mathrm{D}\,x\,\mathrm{e}^{-2x} - 8\,C\,\mathrm{e}^{-2x} + 8\,ys3(x) \tag{3.5}$$

Condiciones iniciales:

```
> e1:=y3(0)=0;
  e2:=D(y3)(0)=1;
  e3:=D(D(y3))(0)=3;
  e4:=D(D(D(y3)))(0)=-2;
```

$$e1 := 3 + A + C = 0$$

$$e2 := -A + B - 2\,C + \mathrm{D} = 1$$

$$e3 := -2\,B + 4\,C - 4\,\mathrm{D} = 3$$

$$e4 := 2\,A + 2\,B - 8\,C + 12\,\mathrm{D} = -2 \tag{3.6}$$

```
> solve({e1,e2,e3,e4},{A,B,C,D});
```

$$\left\{A = -\frac{9}{2},\ B = -\frac{5}{2},\ C = \frac{3}{2},\ \mathrm{D} = 2\right\} \tag{3.7}$$

Solución de la EDO con las condicone sparticulares impuestas:

```
> y:=x->3-9/2*exp(-x)*cos(x)-5/2*exp(-x)*sin(x)+3/2*exp(-2*x)
  +2*x*exp(-2*x);
```

$$y := x \rightarrow 3 - \frac{9}{2}\,\mathrm{e}^{-x}\cos(x) - \frac{5}{2}\,\mathrm{e}^{-x}\sin(x) + \frac{3}{2}\,\mathrm{e}^{-2x} + 2\,x\,\mathrm{e}^{-2x} \tag{3.8}$$

Comprobación:

```
> simplify(diff(y(x),x,x,x,x)+6*diff(y(x),x,x,x)+14*diff(y(x),
  x,x)+16*diff(y(x),x)+8*y(x));
```

$$24 \tag{3.9}$$

```
>
```