> $restart : with(plots) :$

>

Ejercicio 1

> $f := (x, y) \rightarrow \frac{(x^5 + y^6)}{x^4 + y^4};$

$$f := (x, y) \rightarrow \frac{x^5 + y^6}{x^4 + y^4} \tag{1.1}$$

Derivadas parciales en el (0,0)

> $limit\left(\frac{(f(0 + h, 0) - 0)}{h}, h = 0\right);$

$$1 \tag{1.2}$$

> $limit\left(\frac{(f(0, 0 + h) - 0)}{h}, h = 0\right);$

$$0 \tag{1.3}$$

>

Diferenciabilidad en el (0,0)

> $limit\left(\frac{(f(h, k) - 0 - 1\cdot(x - 0) - 0\cdot(y - 0))}{(h^2 + k^2)^{\frac{1}{2}}}, \{h = 0, k = 0\}\right);$

$$limit\left(\frac{\frac{h^5 + k^6}{h^4 + k^4} - x}{\sqrt{h^2 + k^2}}, \{h = 0, k = 0\}\right) \tag{1.4}$$

> $limit\left(\frac{(f(r\cdot\cos(\text{theta}), r\cdot\sin(\text{theta})) - \text{r}\cdot\cos(\text{theta}))}{\text{r}}, \text{r} = 0\right);$

$$-\frac{\cos(\theta)\,\sin(\theta)^4}{2\cos(\theta)^4 + 1 - 2\cos(\theta)^2} \tag{1.5}$$

Gradiente de f en el punto (,)

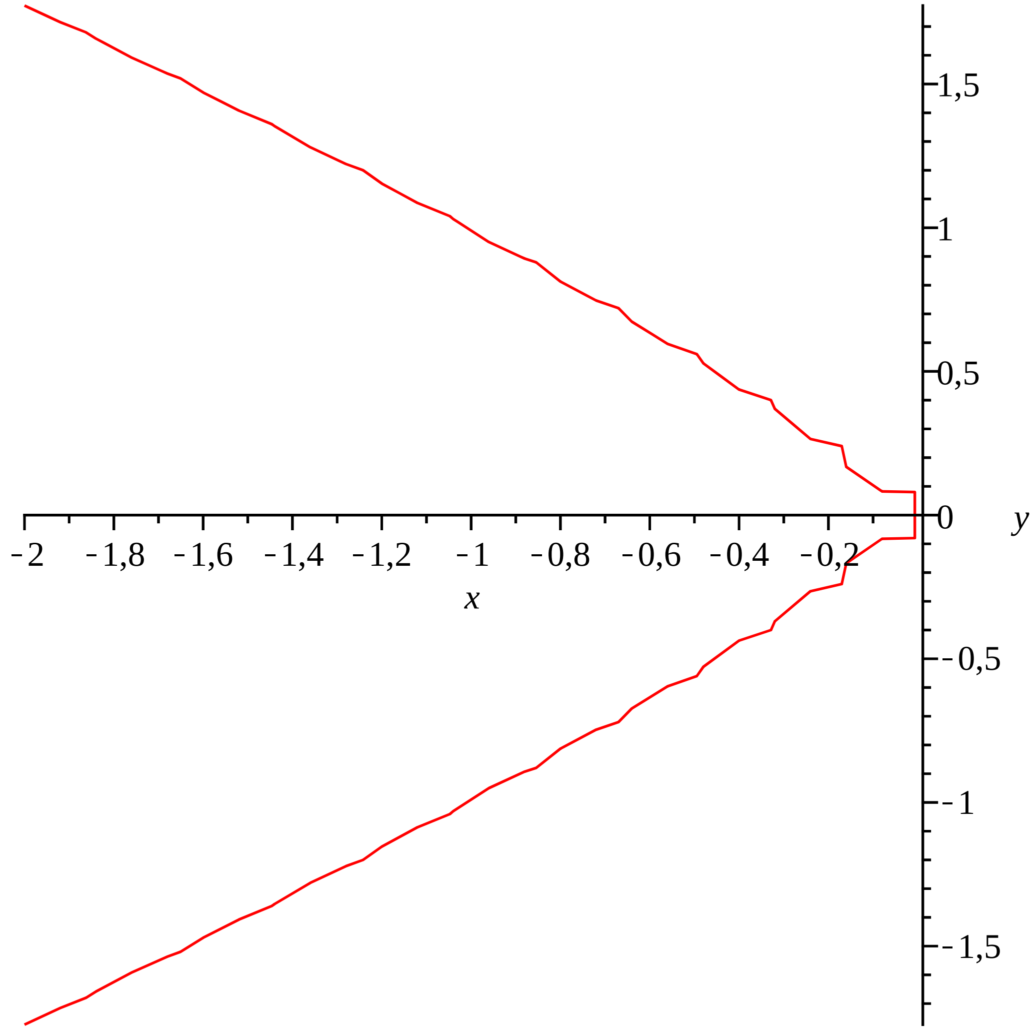
> $D[1](f)(x, y);$
$D[2](f)(x, y);$

$$\frac{5\,x^4}{x^4 + y^4} - \frac{4\,(x^5 + y^6)\,x^3}{(x^4 + y^4)^2}$$
$$\frac{6\,y^5}{x^4 + y^4} - \frac{4\,(x^5 + y^6)\,y^3}{(x^4 + y^4)^2} \tag{1.6}$$

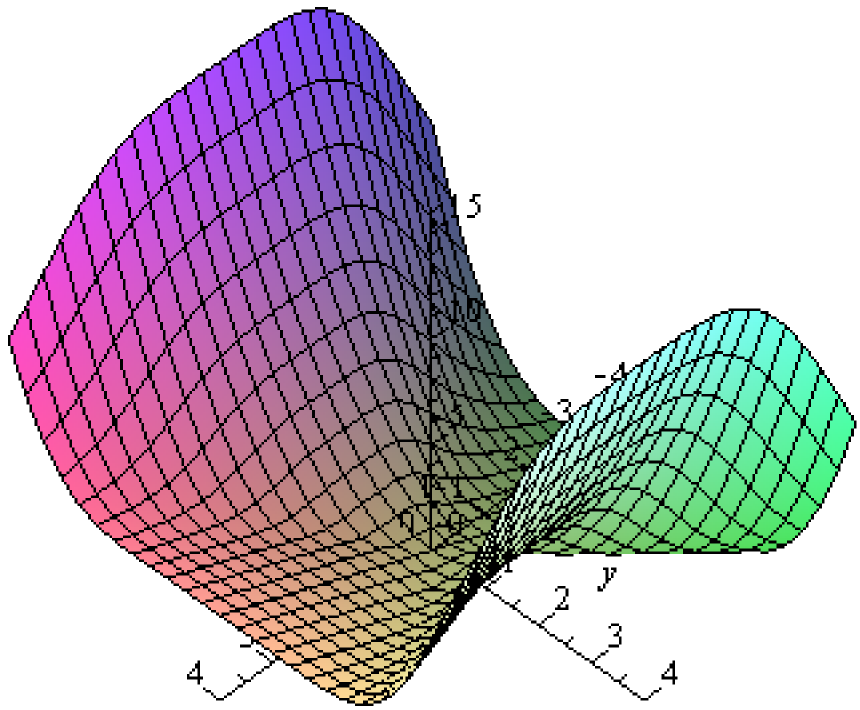
> $D[1](f)(-1, 1);$
$D[2](f)(-1, 1);$

$$\frac{5}{2}$$
$$3 \tag{1.7}$$

> $implicitplot(x^5 + y^6 = 0, x = -2..0, y = -2..2, axes = normal);$



> $plot3d(f(x, y), x=-4..4, y=-4..4, axes=normal);$



Ejercicio 2

> $f := (x, y) \rightarrow 3 \cdot \left(x \cdot y^2\right)^{\left(\frac{1}{3}\right)};$

$$f := (x, y) \rightarrow 3\ \left(x\, y^2\right)^{1/3} \qquad \textbf{(2.1)}$$

> *'Derivadas parciales en el* $(0, 0)$*'*

Derivadas parciales en el $(0, 0)$ **(2.2)**

> $limit\left(\frac{(f(0+h, 0) - f(0, 0))}{h}, h = 0 \right);$

$$0 \qquad \textbf{(2.3)}$$

> $limit\left(\frac{(f(0, 0+h) - f(0, 0))}{h}, h = 0 \right);$

$$0 \qquad \textbf{(2.4)}$$

Diferenciabilidad de f:

> $limit\left(\frac{(f(h,k)-f(0,0)-0\cdot(x-0)-0\cdot(y-0))}{(h^2+k^2)^{\frac{1}{2}}}, \{h=0, k=0\}\right);$

$$limit\left(\frac{3\,(h\,k^2)^{1/3}}{\sqrt{h^2+k^2}}, \{h=0, k=0\}\right) \tag{2.5}$$

> $limit\left(\frac{(f(r\cdot\cos(\text{theta}), r\cdot\sin(\text{theta}))-f(0,0))}{\text{r}}, \text{r}=0\right);$

$$3\,\left(\cos(\theta)\,\sin(\theta)^2\right)^{1/3} \tag{2.6}$$

Gradiente de f en el punto (2,2)

> $simplify(\text{D}[1](f)(x,y));$
> $\text{D}[2](f)(x,y);$

$$\frac{y^2}{(x\,y^2)^{2/3}}$$
$$\frac{2\,x\,y}{(x\,y^2)^{2/3}} \tag{2.7}$$

> $simplify(\text{D}[1](f)(2,2));$
> $simplify(\text{D}[2](f)(2,2));$

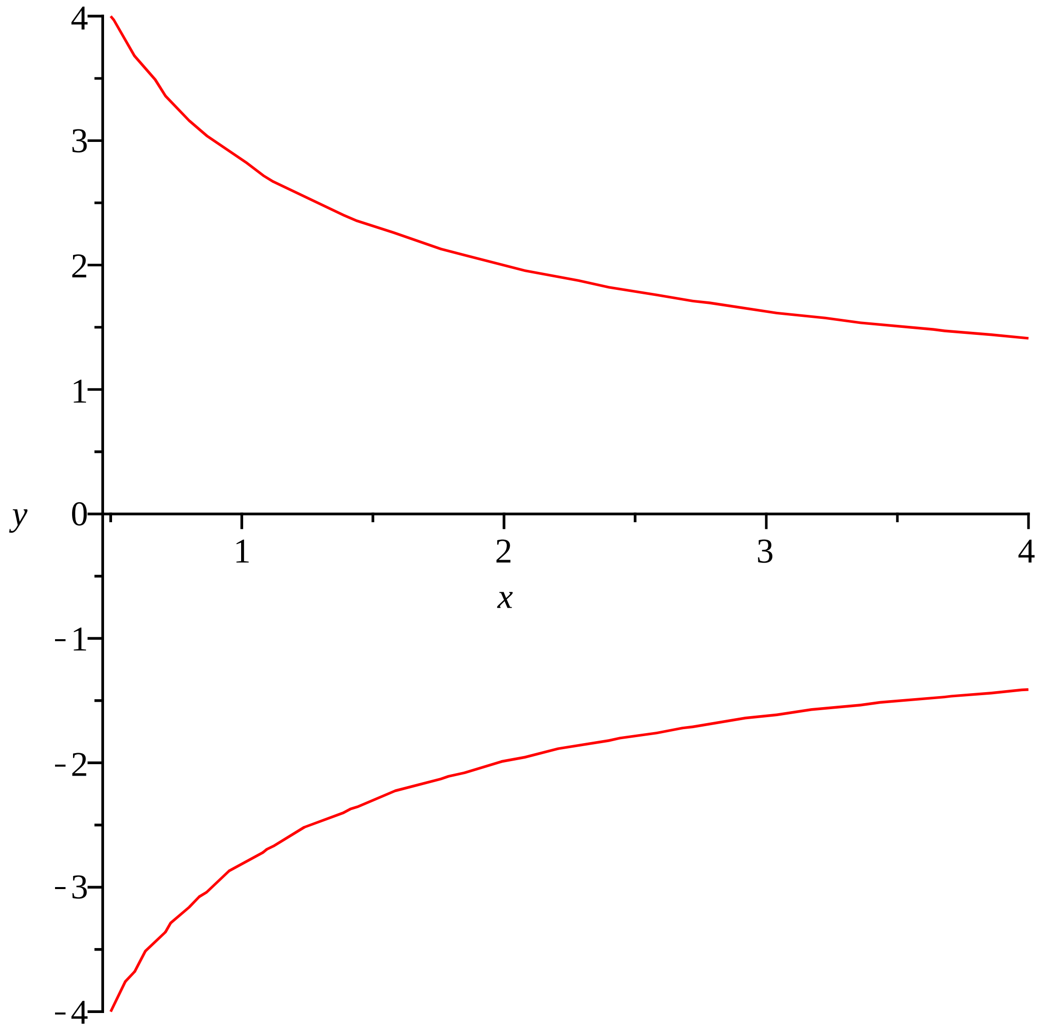
$$1$$
$$2 \tag{2.8}$$

Curva de nivel de f en el punto (2,2)

> $simplify(f(2,2));$

$$6 \tag{2.9}$$

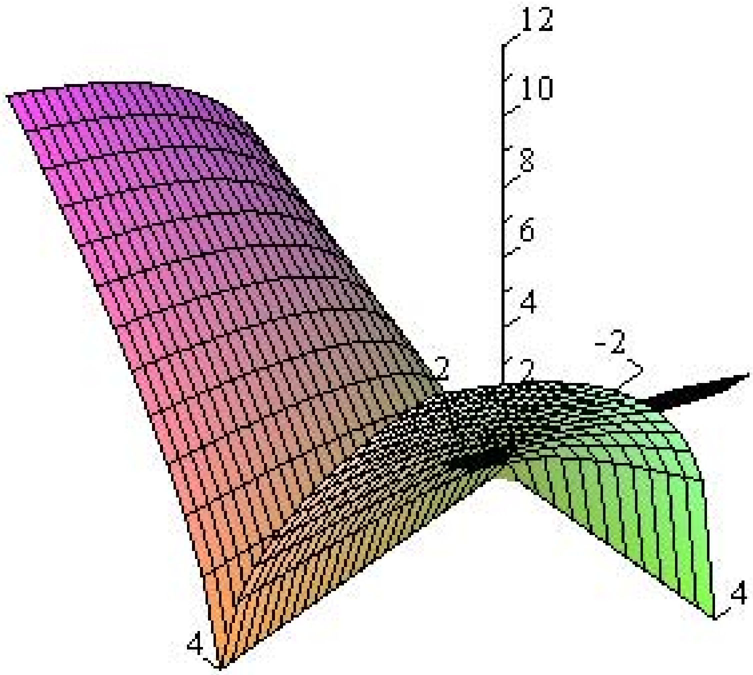
> $implicitplot\left(\text{x}\cdot\text{y}^2=8, x=-4..4, y=-4..4\right);$



> $Gf := plot3d(f(x, y), x = 0 ..4, y = -4 ..4, axes = normal) :$

> $Pf := implicitplot3d(z = 6 + x - 2 + 2 \cdot (y - 2), x = -2 ..2, y = -2 ..2, z = 1 ..3, color = green) :$

> $display(Gf, Pf);$



>

Ejercicio 3

Consideramos la siguiente función :

> $f := (x, y) \rightarrow 2 \cdot x^3 + 2 \cdot y^3 - x^2 - y^2 - 2 \cdot x \cdot y;$

$$f := (x, y) \rightarrow 2\,x^3 + 2\,y^3 - x^2 - y^2 - 2\,x\,y \tag{3.1}$$

Hallar los puntos críticos de f y clasificarlos.

>

> $\mathrm{D}[1](f)(x, y);$
> $\mathrm{D}[2](f)(x, y);$

$$6\,x^2 - 2\,x - 2\,y$$
$$6\,y^2 - 2\,y - 2\,x \tag{3.2}$$

> $solve(\{\mathrm{D}[1](f)(x, y), \mathrm{D}[2](f)(x, y)\}, \{x, y\});$

$$\{x = 0, y = 0\}, \{x = 0, y = 0\}, \{x = 0, y = 0\}, \left\{x = \frac{2}{3}, y = \frac{2}{3}\right\} \tag{3.3}$$

> $\mathrm{D}[1, 1](f)(0, 0);$

> $D[1, 2](f)(0, 0);$
> $D[2, 2](f)(0, 0);$

$$-2$$
$$-2$$
$$-2 \quad \textbf{(3.4)}$$

> $D[1, 1](f)\left(\frac{2}{3}, \frac{2}{3}\right);$
> $D[1, 2](f)\left(\frac{2}{3}, \frac{2}{3}\right);$
> $D[2, 2](f)\left(\frac{2}{3}, \frac{2}{3}\right);$

$$6$$
$$-2$$
$$6 \quad \textbf{(3.5)}$$

> $f(0.5, 0.5);$
> $f(0.8, -0.5);$
> $f(0, 0);$

$$-0.50$$
$$0.684$$
$$0 \quad \textbf{(3.6)}$$

Ejercicio 4

Consideramos la siguiente función :

> $f := (x, y) \rightarrow x^4 + y^3 - 2 \cdot x^2 + 3 \cdot y^2;$

$$f := (x, y) \rightarrow x^4 + y^3 - 2\,x^2 + 3\,y^2 \quad \textbf{(4.1)}$$

Hallar los puntos críticos de f y clasificarlos

>

> $D[1](f)(x, y);$
> $D[2](f)(x, y);$

$$4\,x^3 - 4\,x$$
$$3\,y^2 + 6\,y \quad \textbf{(4.1.1)}$$

> $solve(\{D[1](f)(x, y), D[2](f)(x, y)\}, \{x, y\});$

$$\{x = 0, y = 0\}, \{x = 1, y = 0\}, \{x = -1, y = 0\}, \{x = 0, y = -2\}, \{x = 1, y = -2\}, \{x = -1, y = -2\} \quad \textbf{(4.1.2)}$$

> $D[1, 1](f)(0, 0);$
> $D[1, 2](f)(0, 0);$
> $D[2, 2](f)(0, 0);$

$$-4$$
$$0$$
$$6 \quad \textbf{(4.1.3)}$$

> $D[1, 1](f)(0, -2);$
> $D[1, 2](f)(0, -2);$
> $D[2, 2](f)(0, -2);$

$$-4$$
$$0$$
$$-6 \tag{4.1.4}$$

```
> D[1, 1](f)(1, 0);
  D[1, 2](f)(1, 0);
  D[2, 2](f)(1, 0);
```

$$8$$
$$0$$
$$6 \tag{4.1.5}$$

```
> D[1, 1](f)(-1, 0);
  D[1, 2](f)(-1, 0);
  D[2, 2](f)(-1, 0);
```

$$8$$
$$0$$
$$6 \tag{4.1.6}$$

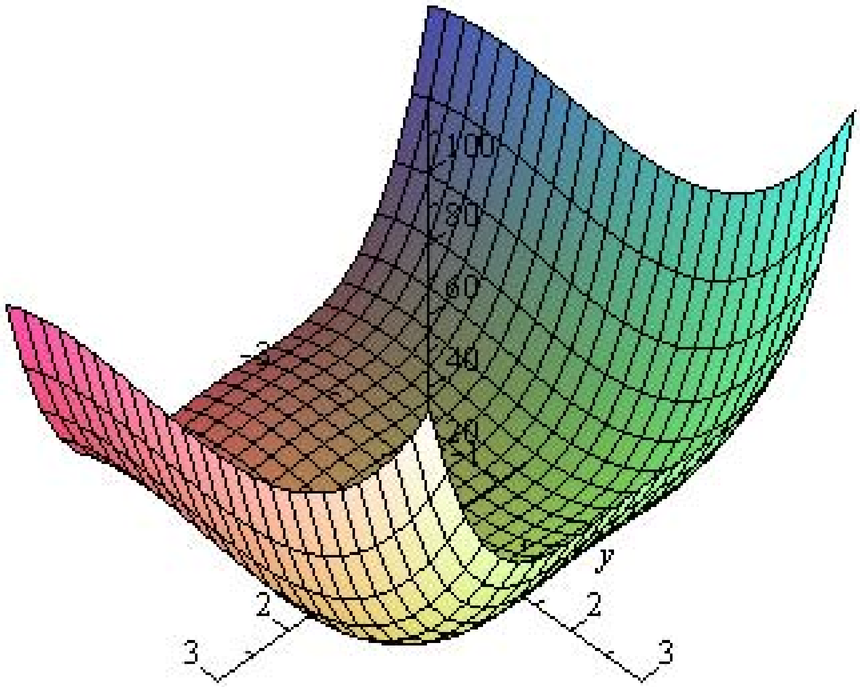
```
> D[1, 1](f)(1,-2);
  D[1, 2](f)(1,-2);
  D[2, 2](f)(1,-2);
```

$$8$$
$$0$$
$$-6 \tag{4.1.7}$$

```
> D[1, 1](f)(-1,-2);
  D[1, 2](f)(-1,-2);
  D[2, 2](f)(-1,-2);
```

$$8$$
$$0$$
$$-6 \tag{4.1.8}$$

```
> plot3d(f(x, y), x =-3 ..3, y =-3 ..3, axes = normal);
```



Hallar los extremos de f sujetos a la restricción $4\cdot x^2+3\cdot y^2=9$

> $g := (x,y) \rightarrow 4\cdot x^2+3\cdot y^2;$

$$g := (x,y) \rightarrow 4\,x^2+3\,y^2 \tag{4.2.1}$$

> D[1](f)(x, y) − lambda·D[1](g)(x, y);
> D[2](f)(x, y) − lambda·D[2](g)(x, y);
> g(x, y) = 9;

$$4\,x^3-4\,x-8\,\lambda\,x$$

$$3\,y^2+6\,y-6\,\lambda\,y$$

$$4\,x^2+3\,y^2=9 \tag{4.2.2}$$

> $solve(\{4\,x^3-4\,x-8\,\lambda\,x,\ 3\,y^2+6\,y-6\,\lambda\,y,\ 4\,x^2+3\,y^2=9\},\ \{x,y,\mathrm{lambda}\});$

$$\left\{x=\frac{3}{2},\ y=0,\ \lambda=\frac{5}{8}\right\},\ \left\{x=-\frac{3}{2},\ y=0,\ \lambda=\frac{5}{8}\right\},\ \left\{x=0,\ y=RootOf\left(-3+\_Z^2,\ label\right.\right. \tag{4.2.3}$$

$$\left.=\_L1\right),\ \lambda=\frac{1}{2}\ RootOf\left(-3+\_Z^2,\ label=\_L1\right)+1\Big\},\ \Big\{x=RootOf\Big(-RootOf\Big($$

$-14\_Z+18+3\_Z^2, label=\_L2)+\_Z^2, label=\_L3), y=-3+RootOf(-14\_Z$
$+18+3\_Z^2, label=\_L2), \lambda=-\frac{1}{2}+\frac{1}{2}RootOf(-14\_Z+18+3\_Z^2, label$
$=\_L2)\}$

> solve(-3 + _Z^2);

$$\sqrt{3}, -\sqrt{3} \quad (4.2.4)$$

> solve(-14 _Z + 18 + 3 _Z^2); #Número complejo

$$\frac{7}{3}+\frac{1}{3}I\sqrt{5}, \frac{7}{3}-\frac{1}{3}I\sqrt{5} \quad (4.2.5)$$

Los puntos críticos son:

> $\left[\frac{3}{2},0\right]; \left[-\frac{3}{2},0\right]; [0,\sqrt{3}]; [0,-\sqrt{3}];$

$$\left[\frac{3}{2},0\right]$$
$$\left[-\frac{3}{2},0\right]$$
$$[0,\sqrt{3}]$$
$$[0,-\sqrt{3}] \quad (4.2.6)$$

> $f\left(\frac{3}{2},0\right);$
$f\left(-\frac{3}{2},0\right);$
$f(0,\sqrt{3});$
$f(0,-\sqrt{3});$

$$\frac{9}{16}$$
$$\frac{9}{16}$$
$$9+3\sqrt{3}$$
$$9-3\sqrt{3} \quad (4.2.7)$$

> $evalf\left(f\left(\frac{3}{2},0\right)\right);$
$evalf(f(0,\sqrt{3}));$
$evalf(f(0,-\sqrt{3}));$

0.5625000000
14.19615242
3.803847576 (4.2.8)

> $\#\left[\frac{3}{2},0\right]; \left[-\frac{3}{2},0\right];$ #son mínimos condicionados;
$\#[0,\sqrt{3}]$ *es un máximo condicionado*