```
> restart:with(linalg):
```

Ejercicio 1

La matriz del sistema de EDOs es:

```
> M:=matrix(2,2,[[1,1],[4,-2]]);
```

$$M := \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \quad (1.1)$$

```
> eigenvectors(M);
```

$$\left[-3, 1, \left\{\begin{bmatrix} 1 & -4 \end{bmatrix}\right\}\right], \left[2, 1, \left\{\begin{bmatrix} 1 & 1 \end{bmatrix}\right\}\right] \quad (1.2)$$

Sistema fundamental de soluciones:

```
> v1:=t->[1,-4]*exp(-3*t);
```

$$v1 := t \rightarrow [1, -4]\, e^{-3t} \quad (1.3)$$

```
> v2:=t->[1,1]*exp(2*t);
```

$$v2 := t \rightarrow [1, 1]\, e^{2t} \quad (1.4)$$

Solución general:

```
> r:=t->C[1]*v1(t)+C[2]*v2(t);
```

$$r := t \rightarrow C_1\, v1(t) + C_2\, v2(t) \quad (1.5)$$

Comprobación:

```
> evalm(diff(r(t),t)-M&*r(t));
```

$$\begin{bmatrix} 0 & 0 \end{bmatrix} \quad (1.6)$$

Ejercicio 2

```
> restart:with(linalg):
```

La matriz del sistema de EDOs es:

```
> M:=matrix(3,3,[[-4,1,1],[1,5,-1],[0,1,-3]]);
```

$$M := \begin{bmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{bmatrix} \quad (2.1)$$

```
> eigenvectors(M);
```

$$\left[-3, 1, \left\{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}\right\}\right], \left[5, 1, \left\{\begin{bmatrix} 1 & 8 & 1 \end{bmatrix}\right\}\right], \left[-4, 1, \left\{\begin{bmatrix} 10 & -1 & 1 \end{bmatrix}\right\}\right] \quad (2.2)$$

Sistema fundamental de soluciones:

```
> v1:=t->[10,-1,1]*exp(-4*t);
```

$$v1 := t \rightarrow [10, -1, 1]\, e^{-4t} \quad (2.3)$$

```
> v2:=t->[1,8,1]*exp(5*t);
```

$$v2 := t \rightarrow [1, 8, 1]\, e^{5t} \quad (2.4)$$

```
> v3:=t->[1,0,1]*exp(-3*t);
```

$$v3 := t \rightarrow [1, 0, 1]\, e^{-3t} \quad (2.5)$$

Solución general:

```
> r:=t->C1*v1(t)+C2*v2(t)+C3*v3(t);
```

$$r := t \to C1\ v1(t) + C2\ v2(t) + C3\ v3(t) \qquad (2.6)$$

Comprobación:

```
> evalm(diff(r(t),t)-M&*r(t));
```

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \qquad (2.7)$$

Solución particular tal que x(0)=4,y(0)=3,z(0)=3.

```
> evalm(r(0)-[4,3,3]);
```

$$\begin{bmatrix} 10\ C1 + C2 + C3 - 4 & -C1 + 8\ C2 - 3 & C1 + C2 + C3 - 3 \end{bmatrix} \qquad (2.8)$$

```
> S1:=convert(evalm(r(0)-[4,3,3]),set);
```

$$S1 := \{-C1 + 8\ C2 - 3,\ C1 + C2 + C3 - 3,\ 10\ C1 + C2 + C3 - 4\} \qquad (2.9)$$

```
> solve(S1,{C1,C2,C3});
```

$$\left\{C1 = \frac{1}{9},\ C2 = \frac{7}{18},\ C3 = \frac{5}{2}\right\} \qquad (2.10)$$

La solución particular es

```
> rp:=t->1/9*v1(t)+7/18*v2(t)+5/2*v3(t);
```

$$rp := t \to \frac{1}{9}\ v1(t) + \frac{7}{18}\ v2(t) + \frac{5}{2}\ v3(t) \qquad (2.11)$$

Comprobación:

```
> evalm(diff(rp(t),t)-M&*rp(t));
  rp(0);
```

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$[4, 3, 3] \qquad (2.12)$$

Ejercicio 3

```
> restart:with(linalg):
```

La matriz del sistema de EDOs es:

```
> M:=matrix(3,3,[[-3,1,0],[2,-4,0],[0,0,-5]]);
```

$$M := \begin{bmatrix} -3 & 1 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix} \qquad (3.1)$$

```
> eigenvectors(M);
```

$$\left[-2, 1, \left\{\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}\right\}\right], \left[-5, 2, \left\{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 & 0 \end{bmatrix}\right\}\right] \qquad (3.2)$$

Sistema fundamental de soluciones:

```
> v1:=t->[1,1,0]*exp(-2*t);
```

$$v1 := t \to [1, 1, 0]\ e^{-2\ t} \qquad (3.3)$$

```
> v2:=t->[0,0,1]*exp(-5*t);
```

$$v2 := t \to [0, 0, 1]\ e^{-5\ t} \qquad (3.4)$$

```
> v3:=t->[1,-2,0]*exp(-5*t);
```

$$v3 := t \to [1, -2, 0]\ e^{-5\ t} \qquad (3.5)$$

Solución general del sistema homogéneo asociado:

```
> rh:=t->C1*v1(t)+C2*v2(t)+C3*v3(t);
```

(3.6)

$$rh := t \to C1\ v1(t) + C2\ v2(t) + C3\ v3(t) \tag{3.6}$$

Comprobación:

```
> evalm(diff(rh(t),t)-M&*rh(t));
```

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{3.7}$$

Solución particular del homogéneo tal que x(0)=1,y(0)=2,z(0)=3:

```
> evalm(r(0)-[1,2,3]);
```

$$\begin{bmatrix} -1 + r(0) & -2 + r(0) & -3 + r(0) \end{bmatrix} \tag{3.8}$$

```
> solve(convert(evalm(r(0)-[1,2,3]),set),{C1,C2,C3});
> rhp:=t->-2/9*v1(t)+2/9*v2(t)+3*v3(t);
```

$$rhp := t \to -\frac{2}{9}\ v1(t) + \frac{2}{9}\ v2(t) + 3\ v3(t) \tag{3.9}$$

```
> evalm(diff(rhp(t),t)-M&*rhp(t));
  rhp(0);
```

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\left[\frac{25}{9}, -\frac{56}{9}, \frac{2}{9}\right] \tag{3.10}$$

Matriz fundamental

```
> concat(v1(t),v2(t),v3(t));
```

$$\begin{bmatrix} e^{-2t} & 0 & e^{-5t} \\ e^{-2t} & 0 & -2\,e^{-5t} \\ 0 & e^{-5t} & 0 \end{bmatrix} \tag{3.11}$$

```
> Phi:=t->concat(v1(t),v2(t),v3(t));
```

$$\Phi := t \to linalg\text{:-}concat(v1(t), v2(t), v3(t)) \tag{3.12}$$

```
> Phi(t);
```

$$\begin{bmatrix} e^{-2t} & 0 & e^{-5t} \\ e^{-2t} & 0 & -2\,e^{-5t} \\ 0 & e^{-5t} & 0 \end{bmatrix} \tag{3.13}$$

```
> inverse(Phi(t));
```

$$\begin{bmatrix} \frac{2}{3\,e^{-2t}} & \frac{1}{3\,e^{-2t}} & 0 \\ 0 & 0 & \frac{1}{e^{-5t}} \\ \frac{1}{3\,e^{-5t}} & -\frac{1}{3\,e^{-5t}} & 0 \end{bmatrix} \tag{3.14}$$

```
> F:=t->[3*t,exp(-t),t^2-t];
```

$$F := t \to [3\,t, e^{-t}, t^2 - t] \tag{3.15}$$

```
> v:=evalm(inverse(Phi(t))&*F(t));
```

$$v := \left[\begin{array}{ccc} \frac{2\,t}{e^{-2\,t}} + \frac{1}{3}\,\frac{e^{-t}}{e^{-2\,t}} & \frac{t^2 - t}{e^{-5\,t}} & \frac{t}{e^{-5\,t}} - \frac{1}{3}\,\frac{e^{-t}}{e^{-5\,t}} \end{array}\right] \tag{3.16}$$

```
> Int(v[1],t)=int(v[1],t);
  Int(v[2],t)=int(v[2],t);
  Int(v[3],t)=int(v[3],t);
```

$$\int \left(\frac{2\,t}{e^{-2\,t}} + \frac{1}{3}\,\frac{e^{-t}}{e^{-2\,t}}\right) dt = \frac{t}{e^{-2\,t}} - \frac{1}{2\,e^{-2\,t}} + \frac{1}{3}\,e^{t}$$

$$\int \frac{t^2 - t}{e^{-5\,t}}\, dt = \frac{1}{125}\,\frac{7 - 35\,t + 25\,t^2}{e^{-5\,t}}$$

$$\int \left(\frac{t}{e^{-5\,t}} - \frac{1}{3}\,\frac{e^{-t}}{e^{-5\,t}}\right) dt = \frac{1}{5}\,\frac{t}{e^{-5\,t}} - \frac{1}{25\,e^{-5\,t}} - \frac{1}{12}\,\left(e^{t}\right)^4 \tag{3.17}$$

```
> simplify(evalm(Phi(t)&*[int(v[1],t),int(v[2],t),int(v[3],t)])
  );
```

$$\left[\begin{array}{ccc} \frac{6}{5}\,t - \frac{27}{50} + \frac{1}{4}\,e^{-t} & \frac{3}{5}\,t - \frac{21}{50} + \frac{1}{2}\,e^{-t} & \frac{7}{125} - \frac{7}{25}\,t + \frac{1}{5}\,t^2 \end{array}\right] \tag{3.18}$$

```
> rp:=t->[ (6/5)*t-27/50+(1/4)*exp(-t), (3/5)*t-21/50+(1/2)*exp
  (-t),  7/125-(7/25)*t+(1/5)*t^2];
```

$$rp := t \rightarrow \left[\frac{6}{5}\,t - \frac{27}{50} + \frac{1}{4}\,e^{-t}, \frac{3}{5}\,t - \frac{21}{50} + \frac{1}{2}\,e^{-t}, \frac{7}{125} - \frac{7}{25}\,t + \frac{1}{5}\,t^2\right] \tag{3.19}$$

```
> evalm(diff(rp(t),t)-M&*rp(t)-F(t));
```

$$\left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right] \tag{3.20}$$

Otro camino para encontrar la solución particular del sistema:

```
> evalm(inverse(Phi(t))&*F(t));
```

$$\left[\begin{array}{ccc} \frac{2\,t}{e^{-2\,t}} + \frac{1}{3}\,\frac{e^{-t}}{e^{-2\,t}} & \frac{t^2 - t}{e^{-5\,t}} & \frac{t}{e^{-5\,t}} - \frac{1}{3}\,\frac{e^{-t}}{e^{-5\,t}} \end{array}\right] \tag{3.21}$$

```
> rp2:=t->simplify(evalm(Phi(t)&*map(int,evalm(inverse(Phi(t))
  &*F(t)),t)));
```

$$rp2 := t \rightarrow simplify(evalm(\Phi(t)\ \&*\ map(int, evalm(linalg\text{:-}inverse(\Phi(t))\ \&*\ F(t)), t))) \tag{3.22}$$

```
> rp2(t);
```

$$\left[\begin{array}{ccc} \frac{6}{5}\,t - \frac{27}{50} + \frac{1}{4}\,e^{-t} & \frac{3}{5}\,t - \frac{21}{50} + \frac{1}{2}\,e^{-t} & \frac{7}{125} - \frac{7}{25}\,t + \frac{1}{5}\,t^2 \end{array}\right] \tag{3.23}$$

```
> evalm(map(diff,rp2(t),t)-M&*rp2(t)-F(t));
```

$$\left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right] \tag{3.24}$$

Solución general del sistema:

```
> r:=t->evalm(rh(t)+rp(t));
```

$$r := t \rightarrow evalm(rh(t) + rp(t)) \tag{3.25}$$

```
> evalm(map(diff,r(t),t)-M&*r(t)-F(t));
```

$$\left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right] \tag{3.26}$$

Ejercico 4

```
> restart:with(linalg):
> M:=matrix(4,4,[[1,1,0,0],[4,-2,0,0],[0,0,-3,1],[0,0,2,-4]]);
```

$$M := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 2 & -4 \end{bmatrix} \tag{4.1}$$

```
> eigenvectors(M);
```

$$\left[-5, 1, \left\{\left[\begin{array}{cccc} 0 & 0 & 1 & -2 \end{array}\right]\right\}\right], \left[-2, 1, \left\{\left[\begin{array}{cccc} 0 & 0 & 1 & 1 \end{array}\right]\right\}\right], \left[-3, 1, \left\{\left[\begin{array}{cccc} 1 & -4 & 0 & 0 \end{array}\right]\right\}\right], \left[2, 1, \left\{\left[\begin{array}{cccc} 1 & 1 & 0 & 0 \end{array}\right]\right\}\right] \tag{4.2}$$

```
> r:=t->[0,0,1,1]*exp(-2*t)*C1+[1,-4,0,0]*exp(-3*t)*C2+[1,1,0,
  0]*exp(2*t)*C3+[0,0,1,-2]*exp(-5*t)*C4;
```

$$r := t \rightarrow [0, 0, 1, 1]\, \mathrm{e}^{-2\,t}\, C1 + [1, -4, 0, 0]\, \mathrm{e}^{-3\,t}\, C2 + [1, 1, 0, 0]\, \mathrm{e}^{2\,t}\, C3 + [0, 0, 1, -2]\, \mathrm{e}^{-5\,t}\, C4 \tag{4.3}$$

```
> evalm(diff(r(t),t)-M&*r(t));
```

$$\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \end{array}\right] \tag{4.4}$$

```
>
```